Solving Differential Equations

If $\frac{dy}{dx} = x^2 - 6x^0$ then you can integrate

$y = \int x^2 - 6 \, dx$ ← state what you intend to do

to find $y = \frac{1}{3}x^3 - 6x + c$ and if you also

know that $x = -3$ when $y = 7$ then

substitute $7 = \frac{1}{3}(-3)^3 - 6 \times -3 + c$

$7 = -9 + 18 + c$

to find $c = -2$

so $\underline{y = \frac{1}{3}x^3 - 6x - 2}$

A curve has equation $y = f(x)$.

$f'(x) = \frac{2}{\sqrt{x}}$ and $f\left(\frac{1}{4}\right) = 3$ $\left(\frac{1}{4}, 3\right)$

$y = \int \frac{2}{\sqrt{x}} \, dx = \int 2x^{-1/2} \, dx = 4x^{1/2} + c$

Substitute $3 = 4 \times \left(\frac{1}{4}\right)^{1/2} + c$

$3 = 2 + c, \quad c = 1$

$y = 4x^{1/2} + 1$

$\underline{y = 4\sqrt{x} + 1}$

A curve has equation $y = g(x)$.

$\frac{dy}{dx} = 6\cos 3x$ and the point $\left(\frac{\pi}{2}, 3\right)$ is on the curve

Find an expression for $g(x)$.

$g(x) = y = \int 6\cos 3x \, dx = \frac{6}{3}\sin 3x = 2\sin 3x + c$

Substitute $3 = 2\sin\left(3\times\frac{\pi}{2}\right) + c$

$= 2\times -1 + c$

$c = 5$

$\underline{g(x) = 2\sin 3x + 5}$

Textbook
Page 280

Exercise 11G

1 $x^3 - x^2 + 4x + 25$

2 $4x^{\frac{3}{2}} - 24$

3 $3 - 2\cos 2x$

4 **a** $x^3 + 5x^2 - 2x - 24$

b **i** $(-3)^3 + 5(-3)^2 - 2(-3) - 24 = 0$

ii $(x - 2)(x + 4)(x + 3)$

c $(2, 0)$

$(-4, 0)$

$(-3, 0)$

5 **a** $4\left(\frac{x^2}{2} - x\right) + 7 = 2(x - 1)^2 + 5$

b By completing the suare you can see that the minimum value is 5 so no roots.

6 **a** $x^3 + x^2 - 5x + 3$

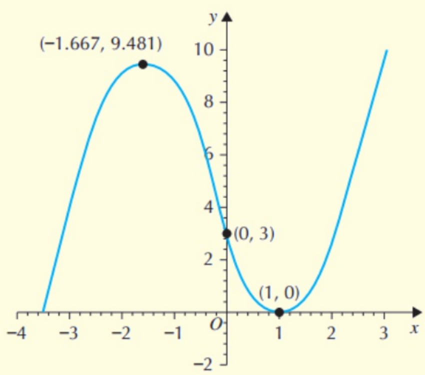
b **i** $(-3)^3 + (-3)^2 - 5(-3) + 3 = 0$

ii $x = 1$

c **i** **ii** (1, 0) minimum

(−1.667, 9.481) maximum

d



7 **a** $x^3 + 2x^2 - 4x - 8$

b **i** $(2)^3 + 2(2)^2 - 4(2) - 8 = 0$

ii $(x + 2)(x + 2)(x - 2)$

c t.p. are at $x = -2$ and $x = \frac{2}{3}$.

Since $x = -2$ is both a root $(-2, 0)$ and a turning point the x-axis must be a tangent at $x = -2$.

8 **a** $\frac{2}{3}$

b $y = mx + c$

$f'(x) = \frac{2}{3}x - 4$

c $= \frac{1}{3}x^2 - 4x + 15$

9 **a** $-2x + 6$

b $-x^2 + 6x - 9$