Higher Hw 22 Solutions

1. $y = 5x + 2$

$x = 5y + 2$ (swap)

$5y = x - 2$ (change subject)

$y = \frac{x-2}{5} = f^{-1}(x)$ 3

2. $y - b = m(x - a)$

$y - 4 = -4(x - 3)$

- gradient $m = \frac{5-3}{7-(-1)} = \frac{1}{4}$ $m_\perp = -4$
- coord (a, b) $M_{PQ} = \left(\frac{7-1}{2}, \frac{5-3}{2}\right) = (3, 4)$ 4

3a) S — T — U, ST : TU = 2 : 1

$t = \frac{1}{3}\left[\begin{pmatrix}5\\2\\1\end{pmatrix} + 2\begin{pmatrix}3\\5\\4\end{pmatrix}\right]$

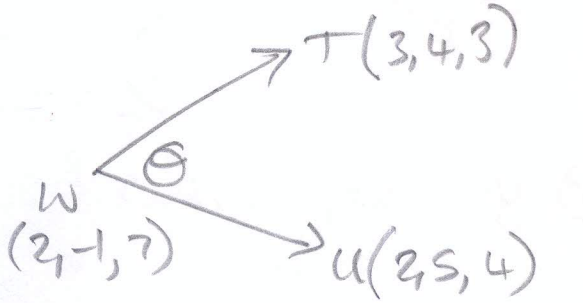
$= \frac{1}{3}\begin{pmatrix}9\\12\\9\end{pmatrix} = \begin{pmatrix}3\\4\\3\end{pmatrix}$

so T is $(3, 4, 3)$ 2

b) $\cos\theta = \frac{\vec{WT} \cdot \vec{WU}}{|WT||WU|}$

$= \left(\frac{42}{\sqrt{42}\sqrt{45}}\right)$

$\theta = 14.96°$



$\vec{WT} = \begin{pmatrix}3\\4\\3\end{pmatrix} - \begin{pmatrix}2\\-1\\7\end{pmatrix} = \begin{pmatrix}1\\5\\-4\end{pmatrix}$ $|WT| = \sqrt{42}$

$\vec{WU} = \begin{pmatrix}2\\5\\4\end{pmatrix} - \begin{pmatrix}2\\-1\\7\end{pmatrix} = \begin{pmatrix}0\\6\\-3\end{pmatrix}$ $|WU| = \sqrt{45}$

4

4. a) $f(x) = (2x+3)^4$

$f'(x) = 4(2x+3)^3 \times 2$

$= \underline{8(2x+3)^3}$ 2

b) $f(x) = 5\cos x$

$f'(x) = \underline{-5\sin x}$ 2

5. a) If $(x-1)$ is a factor $f(1) = 0$

1	1	0	−3	2
	↓	1	1	−2
	1	1	−2	0 remainder

$f(1) = 0 \therefore (x-1)$ is a factor of $f(x)$.

b) $f(x) = x^3 - 3x + 2$

$= (x-1)(x^2+x-2)$

$= (x-1)(x+2)(x-1)$

c) $f(x) = (x-1)(x+2)(x-1) = 0$

$x = 1, x = -2$

d) Stationary points when

$m = \frac{dy}{dx} = 0$

$\frac{dy}{dx} = 3x^2 - 3 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

when $x = 1$, $y = (1)^3 - 3(1) + 2$

$(1, 0) \quad = 0$

when $x = -1$, $y = (-1)^3 - 3(-1) + 2$

$(-1, 4) \quad = 4$

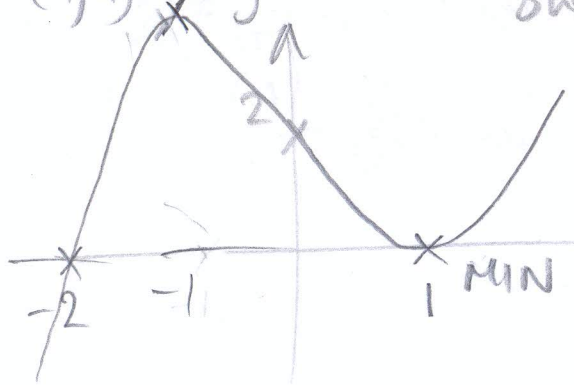
e) y intercept when $x = 0$

$y = 0^3 - 3\times0 + 2 = 2$

$(-1, 4)$ MAX

x	−2	−1	0	1	2
$\frac{dy}{dx}$	+	0	−	0	+
Shape	/	—	\	—	/

5



e)

- Shape
- annotation 2