Factorising With Fractions

Show that $(2x-1)$ is a factor of $f(x) = 2x^3 + 11x^2 + 4x - 5$ then express $f(x)$ in a factorised form.

$$f(x) = 2x^3 + 11x^2 + 4x - 5$$

quotient

$$= \left(x - \tfrac{1}{2}\right)\left(2x^2 + 12x + 10\right)$$

factor

$$= (2x-1)(x^2 + 6x + 5)$$

$$= (2x-1)(x+5)(x+1)$$

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & 11 & 4 & -5 \\ & \downarrow & 1 & 6 & 5 \\ \hline & 2 & 12 & 10 & \underline{\underline{0}} \text{ remainder} \end{array}$$

$f\left(\tfrac{1}{2}\right) = 0 \therefore \left(x - \tfrac{1}{2}\right)$ is a factor

Find the quotient and the remainder for dividing $3x^3 + 8x^2 + 6x - 5$ by $(3x-1)$.

$$f(x) = 3x^3 + 8x^2 + 6x - 5$$

$$= \left(x - \tfrac{1}{3}\right)\left(3x^2 + 9x + 9\right)$$

$$= (3x-1)(x^2 + 3x + 3)$$

~~factor~~? quotient

$$\begin{array}{c|cccc} \frac{1}{3} & 3 & 8 & 6 & -5 \\ & \downarrow & 1 & 3 & 3 \\ \hline & 3 & 9 & 9 & \underline{\underline{-2}} \text{ remainder} \end{array}$$

Fully factorise $3x^3+4x^2-5x-2$

given that $(3x+1)$ is a factor.

$$f(x) = 3x^3+4x^2-5x-2$$
$$= \left(x+\tfrac{1}{3}\right)(3x^2+3x-6)$$
$$= (3x+1)(x^2+x-2)$$
$$= (3x+1)(x+2)(x-1)$$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & 4 & -5 & -2 \\ & \downarrow & -1 & -1 & 2 \\ \hline & 3 & 3 & -6 & \underline{\underline{0}} \end{array}$$

remainder

$f(-\tfrac{1}{3}) = 0 \therefore \left(x+\tfrac{1}{3}\right)$ is a factor