

Applications of Logarithmic Functions

The magnitude of an earthquake measured on the Richter Scale is given by:

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Example 1.24

$$M = \log_{10} \left(\frac{I}{I_0} \right) \quad M_s = 2.9$$

M - magnitude
 I - intensity
 I_0 - constant

$$M_s = \log_{10} \left(\frac{I}{I_0} \right) = 2.9$$

What is the magnitude of the earthquake in Wales?

$$\begin{aligned} M_w &= \log_{10} \left(\frac{8 \times I}{I_0} \right) \\ &= \log_{10} 8 + \log_{10} \left(\frac{I}{I_0} \right) \\ &= \log_{10} 8 + 2.9 \\ &= \underline{\underline{3.8}} \end{aligned}$$

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pH Scale

1.a) The pH of ethanoic acid can be calculated using the expression:

$$\begin{aligned} \text{pH} &= -\frac{1}{2} \log_{10} K_a + \frac{1}{2} \log_{10} c \\ &= -\frac{1}{2} \log_{10} (1.8 \times 10^{-5}) + \frac{1}{2} \log_{10} 2 \\ &= 2.522878745 \\ &\Rightarrow \underline{\underline{2.52}} \text{ (to 2dp)} \end{aligned}$$

$K_a = 1.8 \times 10^{-5}$
 $c = 2$

$$b) pH = -\frac{1}{2} \log_{10} Ka + \frac{1}{2} \log_{10} c = -\log_{10} H$$

→ H&S
↙ reorder

$$\frac{1}{2} \log_{10} c = -\log_{10} H + \frac{1}{2} \log_{10} Ka$$

$$\times 2 \quad \frac{1}{2} \log_{10} c = \frac{1}{2} \log_{10} Ka - \log_{10} H$$

$$\log_{10} c = \log_{10} Ka - 2 \log_{10} H$$

$$\log_{10} c = \log_{10} Ka - \log_{10} H^2$$

$$\log_{10} c = \log_{10} \left(\frac{Ka}{H^2} \right) \text{ as expected.}$$

1 a 2.523

b $-\log_{10} H = -\frac{1}{2} \log_{10} Ka + \frac{1}{2} \log_{10} c$

$$\log_{10} Ka - 2 \log_{10} H = \log_{10} c$$

$$\log_{10} Ka - \log_{10} H^2 = \log_{10} c$$

$$\log_{10} c = \log \left(\frac{Ka}{H^2} \right)$$

2 50

3 a Square in power of V becomes multiple of 2 since $\log(x^2) = 2 \log x$

$$10 \log_{10} \left(\frac{V_1^2}{V_0^2} \right) = 20 \log_{10} \left(\frac{V_1}{V_0} \right)$$

b 19.95 V

4 0

5 1.5229